JEE MAIN 2023
Paper with Solution

MATHS | 24th Jan 2023 _ Shift-2

Umeed Rank Ki Ho Ya Selection Ki,
JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

61. If, $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3), x\in\mathbb{R}$ then

(1) $f(1)+f(2)+f(3)=f(0)$ (2) $2f(0)-f(1)+f(3)=f(2)$

(3) $3f(1)+f(2)=f(3)$ (4) $f(3)-f(2)=f(1)$

Sol. **2**

$f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$

$f(x) = x^3 - ax^2 + bx - c$

$f'(x) = 3x^2 - 2ax + b$

$f''(x) = 6x - 2a$

$f'''(x) = 6$

$f'''(3) = 6$

$f'(1) = 3 - 2a + b = a \Rightarrow 3a = b + 3$

$f''(2) = 12 - 2a = b \Rightarrow 2a = 12 - b$

$a = 3, b = 6$

$f'''(3) = 6 = c$

$f(x) = x^3 - 3x^2 + 6x - 6$

$f(0) = -6$ $f(2) = 2$

$f(1) = -2$ $f(3) = 12$

62. If the system of equations

$$x + 2y + 3z = 3$$
$$4x + 3y - 4z = 4$$
$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to :

(1) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ (2) $\left(-\frac{72}{5} \cdot -\frac{21}{5}\right)$ (3) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ (4) $\left(\frac{72}{5}, \frac{21}{5}\right)$

Sol. **3**

Planes are not parallel

$\therefore (x + 2y + 3z - 3) + a(4x + 3y - 4z - 4)$

$= 8x + 4y - \lambda z - 9 - \mu = 0$

$$\frac{1+4a}{8}=\frac{2+3a}{4}=\frac{3-4a}{-\lambda}=\frac{-3-4a}{-9-\mu}$$

(i) $1 + 4a = 4 + 6a$

$a=\frac{-3}{2}$

(ii) $\frac{2-\frac{9}{2}}{4}=\frac{3+6}{-\lambda}$

$$-\lambda = \frac{36}{-5} \times 2$$

$$\lambda = \frac{72}{5}$$

(iii) $\frac{-5}{8} = \frac{-3-4a}{-9-\mu}$

$$\frac{5}{8} = \frac{3-6}{-9-\mu}$$

$$-9-\mu = \frac{-24}{5}$$

$$\mu = \frac{-45+24}{5}$$

$$\mu = \frac{-21}{5}$$

63. If, then $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1011 (2) 2010 (3) 1010 (4) 2011

Sol. **1**

$$f(x) = \frac{4^x}{4^x+2}$$

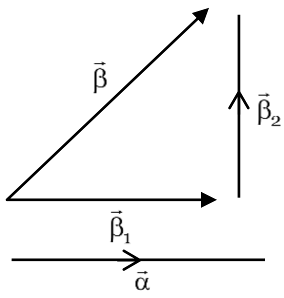
$$f(1-x) = \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4}{4+2.4^x} = \frac{2}{2+4^x}$$

f(x) + f(1–x) = 1

64. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

(1) 7 (2) 9 (3) 6 (4) 11

Sol. **1**



$$\overrightarrow{\beta_1} = \frac{(\vec{\alpha} \cdot \vec{\beta})}{|\vec{\alpha}|}\hat{\alpha}$$

$=\left(\frac{4+6-20}{\sqrt{16+9+25}}\right)\frac{(4,3,5)}{\sqrt{50}}$

$=\frac{-10}{50}(4,3,5)$

$\vec{\beta}_1=\frac{(-4,-3,-5)}{5}$

$\vec{\beta}_1+\vec{\beta}_2-=(1,2,-4)$

$\beta_2=\left(1+\frac{4}{5},2+\frac{3}{5},-4+1\right)$

$\beta_2=\left(\frac{9}{5},\frac{13}{5},-3\right)$

$\therefore 5\beta_2=(9,13,-15)$

$\therefore 5\beta_2\cdot(1,1,1)=9+13-15$

= 7

65. Let $y=y(x)$ be the solution of the differential equation $(x^2-3y^2)dx+3xydy=0, y(1)=1$. Then $6y^2(e)$ is equal to

(1) $2e^2$ (2) $3e^2$ (3) e^2 (4) $\frac{3}{2}e^2$

Sol. **1**

$(x^2-3y^2)dx+3xydy=0$

$\frac{dy}{dx}=\frac{3y^2-x^2}{3xy}$

$y=vx$

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{3v^2x^2-x^2}{3vx^2}$

$v+x\frac{dv}{dx}=\frac{3v^2-1}{3v}$

$\frac{xdy}{dx}=\frac{3v^2-1}{3v}-v\Rightarrow\frac{-1}{3v}$

$3vdv=-\frac{dx}{x}$

$\frac{3v^2}{2}=-\ln x+C$ [y(1) = 1]

$\frac{3y^2}{2x^2}=-\ln x+C$

$C = \frac{3}{2}$

$\frac{3y^2}{2x^2} = \frac{3}{2}\ln e = \ln x$

$\therefore 3y^2 = 3x^2 \ln e - 2x^2 \ln x$

$x = e \quad 3y^2 = 3e^2\ln e - 2e^2\ln e$

$= e^2 \ln e$

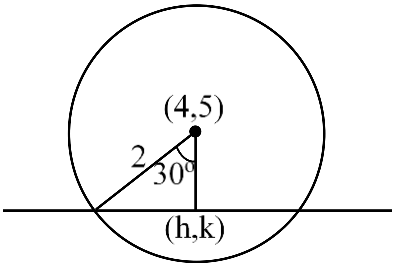
$= e^2$

$6y^2 = 2e^2$

66. The locus of the mid points of the chords of the circle $C_1: (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{3\pi}{4}$

Sol. **2**



$r_1 = 2\cos 30^o$

$r_3 = 2\cos 60^o$

$3 = r_2^2 + 1 \Rightarrow r_2 = \sqrt{2}$

$2\cos\theta = \sqrt{2}, \ \theta = \frac{\pi}{4}$

$\therefore \theta_2 = \frac{\pi}{2}$

67. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 0 (2) 3 (3) 4 (4) 2

Sol. **1**

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left[x + \frac{1}{x}\right] + 5 = 0$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t+1)(t-1) = 0$

$t = 1, t = \frac{-1}{3}$

68. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}\,A))| = 12^4$ Then $|A^{-1}\text{adj}\,A|$ is equal to

(1) $\sqrt{6}$ (2) $2\sqrt{3}$ (3) 12 (4) 1

Sol. **2**

$$|\text{adj}(\text{adj adj}A)| = |A|^{(n-1)^3} = 12^4$$

$$|A|^8 = (12)^4$$

$$|A| = (12)^{\frac{1}{2}}$$

$$\therefore \left|A^{-1} \cdot \text{adj}(A)\right| = \left|A^{-1}\right| \times |\text{adj}A|$$

$$= \frac{1}{|A|} \times |A|^{n-1}$$

$$= \frac{1}{|A|} \times |A|^2 = |A| = \sqrt{12} = 2\sqrt{3}$$

69. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to

(1) 2π (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Sol. **1**

$$48 \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{dx}{\sqrt{9-4x^2}} = \frac{48}{2} \sin^{-1}\left\{\frac{2x}{3}\right\}^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = 2\pi$$

70. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 125 (2) 225 (3) 150 (4) 120

Sol. **4**

$$\begin{bmatrix} \_ & \_ & \_ & \boxed{1} & \_ \\ \_ & \_ & \boxed{1} & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \end{bmatrix} \longrightarrow {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$$

${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 120$

71. If $\left({}^{30}C_1\right)^2+2\left({}^{30}C_2\right)^2+3\left({}^{30}C_3\right)^2+....+30\left({}^{30}C_{30}\right)^2=\frac{\alpha 60!}{(30!)^2}$ then α is equal to :

(1) 30 (2) 10 (3) 60 (4) 15

Sol. **4**

$C_1^2+2C_2^2+3C_3^2...+30C_{30}^2$

$S=0C_0^2+1C_1^2+...+30C_{30}^2$

$S=30C_{30}^2+29C_{29}^2+...+0C_0^2$

$2S=30\left[C_0^2+C_1^2+...+C_{30}^2\right]$

$S=15\times\ {}^{60}C_{30}=\frac{\alpha\cdot 60!}{(30!)^2}\Rightarrow\alpha=15$

72. Let the plane containing the line of intersection of the planes $P1: x+(\lambda+4)y+z=1$ and $P2: 2x+y+z=2$ pass through the points (0,1,0) and (1,0,1). Then the distance of the point $(2\lambda,\lambda,-\lambda)$ from the plane P2 is

(1) $4\sqrt{6}$ (2) $3\sqrt{6}$ (3) $5\sqrt{6}$ (4) $2\sqrt{6}$

Sol. **2**

$[x + (\lambda+4)y + z-1] + \mu[2x+y + z-2] = 0$

(0,1,0)

(i) $(\lambda + 4 - 1) + \mu[-1] = 0$

$\lambda - \mu = -3$

(1,0,1) (ii) $1 + \mu[1] = 0 \Rightarrow \mu = -1, \lambda = -4$

$\therefore$ point $(-8,-4,4)$; $2x + y + z - 2 = 0$

$$d=\left|\frac{-16-4+4-2}{\sqrt{6}}\right|=\frac{18}{\sqrt{6}}=3\sqrt{6}$$

73. Let f(x) be a funciton such that $f(x+y)=f(x)\cdot f(y)$ for all $x, y\in N$. If f(1) = 3 and $\sum_{k=1}^{n}f(k)=3279$, then the value of n is

(1) 9 (2) 6 (3) 8 (4) 7

Sol. **4**

$f(x+y) = f(x).f(y),\ x,y\in N$

$f(2) = 3^2$

$f(3) = 3^3$ $\quad\therefore 3\frac{\left[3^n-1\right]}{2}=3279$

$3^n - 1 = 1093\times 2$

$3^n - 1 = 2186$

$3^n = 2187$

$n = 7$

74. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

(1) 210 (2) 220 (3) 200 (4) 105

Sol. **1**

$a + (a + 2d) = 10 \Rightarrow a + d = 5$ ...(1)

Mean $\Rightarrow \dfrac{\frac{6}{2}[2a+5d]}{6} = \dfrac{19}{2}$

$2a + 5d = 19$ ...(2)

from (1) and (2)

$3d = 9 \Rightarrow d = 3;\ a = 2$

$\therefore\ \sigma^2 = \dfrac{\sum x_i^2}{n} - (\bar{x})^2$

$= \dfrac{2^2+5^2+8^2+11^2+14^2+17^2}{6} - \left(\dfrac{19}{2}\right)^2$

$= \dfrac{699}{6} - \dfrac{361}{4}$

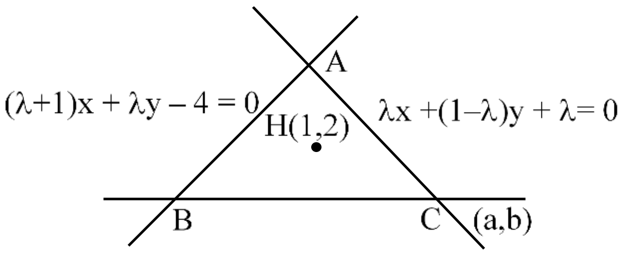
$= \dfrac{233}{2} - \dfrac{361}{4}$

$8\sigma^2 = 932 - 722 = 210$

75. The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x + \lambda y = 4$ and $\lambda x + (1-\lambda)y + \lambda = 0$ respectively. Its vertex A is on the y - axis and its orthocentre is (1,2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) 4 (2) 2 (3) $\sqrt{6}$ (4) $2\sqrt{2}$

Sol. **4**



$(\lambda+1)(1-\lambda)x + \lambda(1-\lambda)y = 4(1-\lambda)$

$\lambda^2 x + \lambda(1-\lambda)y = -\lambda^2$

$-\quad -\qquad\qquad +$

$(1-\lambda^2-\lambda^2)x = 4 - 4\lambda + \lambda^2$

$(1-2\lambda^2)x = 4 - 4\lambda + \lambda^2$

$x = 0 \Rightarrow \lambda = 2$

$\left.\begin{array}{l} AB: 3x+2y=4 \\ AC: 2x-y+2=0 \end{array}\right\} A(0,2)$

$CH \perp AB$

$\left(\frac{b-2}{a-1}\right)\times\left(\frac{-3}{2}\right)=-1$

$3b - 6 = 2a - 2$ Also $2a - b + 2 = 0$

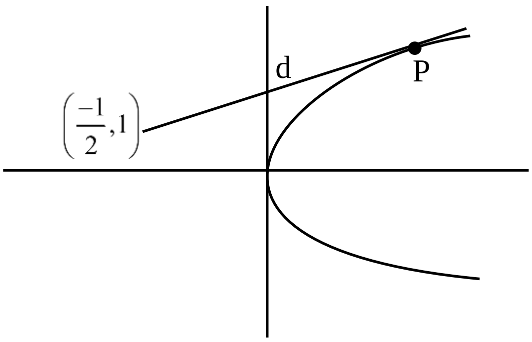
$3b - 2a = 4$ $b = 2a + 2$

$6a + 6 = 2a + 4$ $C\left(-\frac{1}{2},1\right)$

$4a = -2$

$a=\frac{-1}{2}, b=1$

$\therefore\ y^2 = 6x$



$ty = x + \frac{3}{2}t^2$

$t = -\frac{1}{2}+\frac{3}{2}t^2$

$3t^2 - 1 = 2t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t + 1)(t-1) = 0$

$t = 1$

$P\left(\frac{3}{2},3\right)$ $\therefore d=\sqrt{\left(\frac{3}{2}+\frac{1}{2}\right)^2+(3-1)^2}$

$=\sqrt{4+4}=2\sqrt{2}$

76. Let p and q be two statements. Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge q)$ (2) $p \vee (p \wedge (\sim q))$ (3) $(\sim p) \vee q$ (4) $p \vee ((\sim p) \wedge q)$

Sol. **3**

$P \wedge (P \Rightarrow \sim q)$ $\quad\quad$ $P \to q$

$P \wedge (\sim P \vee \sim q)$

$\therefore$ Its negation will be

$\sim P \vee [P \wedge q]$

$= [\sim P \vee P] \wedge [\sim P \vee q]$

$= \sim P \vee q$

77. The set of all values of a for which $\lim_{x \to a}([x-5]-[2x+2])=0$, where $[\propto]$ denotes the greatest integer less than or equal to α is equal to

(1) $[-7.5, -6.5)$ (2) $[-7.5, -6.5]$ (3) $(-7.5, -6.5]$ (4) $(-7.5, -6.5)$

Sol. **4**

$\lim_{x \to \alpha}([x]-5-[2x]-2)=0$

$\lim_{x \to \alpha}([x]-[2x])=7$

$[\alpha]-[2\alpha]=7$

If $\alpha = -7.5$ $\quad$ $[-7.5] = -8$

$[-15] = -15$ $\quad$ $\therefore -8 + 15 = 7$

If $\alpha = -6.5$ $\quad$ $[-6.5] = -7$ $\quad$ $-7 + 13 = 6$

$[-13] = -13$

$\therefore \quad \alpha \in (-7.5, -6.5)$

78. If the foot of the perpendicular drawn from (1,9,7) to the line passing through the point (3,2,1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 3 (2) 1 (3) -1 (4) 5

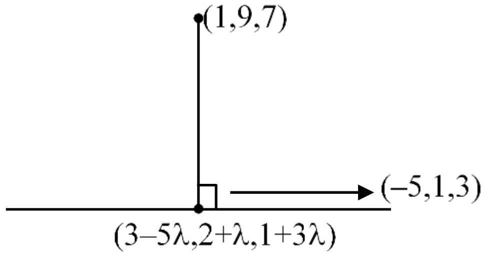
Sol. **4**

$\bar{n}_1 = (1,2,1)\bar{n}_2 = (0,3,-1)$

$$\bar{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$= (-5, 1, 3)$

$$\therefore \text{line}: \frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = \lambda$$



$(2-5\lambda).-5+(\lambda-7).1+(3\lambda-6).3=0$

$25\lambda-10+\lambda-7+9\lambda-18=0$

$35\lambda=35$

$\lambda=1$

$\therefore$ Point is $(-2,3,4)$ $\quad \alpha+\beta+\gamma=5$

79. The number of integers, greater than 7000 that can be formed, using the digits 3,5,6,7,8 without repetition, is

(1) 168 (2) 220 (3) 120 (4) 48

Sol. **1**

C-1 □ □ □ □

$2\times4\times3\times2=48$

C-2 5!(5 digit nos) = $\dfrac{120}{168}$

80. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $-\frac{1}{2}(\sqrt{3}-i)$ (2) $-\frac{1}{2}(1-i\sqrt{3})$ (3) $\frac{1}{2}(1-i\sqrt{3})$ (4) $\frac{1}{2}(\sqrt{3}+i)$

Sol. **1**

$\frac{\pi}{2}-\frac{2\pi}{9}$

$=\frac{95-4\pi}{18}=\frac{5\pi}{18}$

$\Rightarrow \dfrac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}$

$=\dfrac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}} \Rightarrow \left(\dfrac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$

$=e^{i\left(\frac{5\pi}{18}\right)^3} \quad =e^{i\left(\frac{5\pi}{6}\right)}$

$\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{2}+\frac{i}{2}$

SECTION B

81. If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is

Sol. **384**

$P\left(-\sqrt{6},\sqrt{6},\sqrt{6}\right)$ $\qquad Q\left(\lambda,2\sqrt{6},-2\sqrt{6}\right)$

$\bar{n}_1=(2,3,4)$ $\qquad \bar{n}_2=(3,4,5)$

$$\bar{n}_1\times\bar{n}_2\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\3&4&5\end{vmatrix}=\hat{i}(-1)-\hat{j}(-2)+\hat{k}(-1)$$

$= (-1, 2, -1)$

$$\therefore S_d\left|\frac{\overline{PQ}\cdot(-1,2,-1)}{\sqrt{6}}\right|=\frac{\left(\lambda+\sqrt{6},\sqrt{6},-3\sqrt{6}\right)\cdot(-1,2,-1)}{\sqrt{6}}$$

$$=\left|\frac{-\lambda-\sqrt{6}+2\sqrt{6}+3\sqrt{6}}{\sqrt{6}}\right|=6$$

$$\Rightarrow\left|-\lambda+4\sqrt{6}\right|=6\sqrt{6}$$

$(+)\qquad -\lambda+4\sqrt{6}=6\sqrt{6}$ $\qquad\qquad (-)\qquad \lambda-4\sqrt{6}=6\sqrt{6}$

$\lambda=-2\sqrt{6}$ $\qquad\qquad \lambda=10\sqrt{6}$

$$\therefore\left(8\sqrt{6}\right)^2=384$$

82. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2=\lambda x$ with one vertex at the vertex of the parabola, is

Sol. 432

A	B	C
4R 6B	5R 5B	4R 4B

$$\text{P(Red from C)}=\frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\cdot\frac{\lambda}{\lambda+4}+\frac{1}{3}\cdot\frac{4}{10}+\frac{1}{3}\cdot\frac{5}{10}}$$

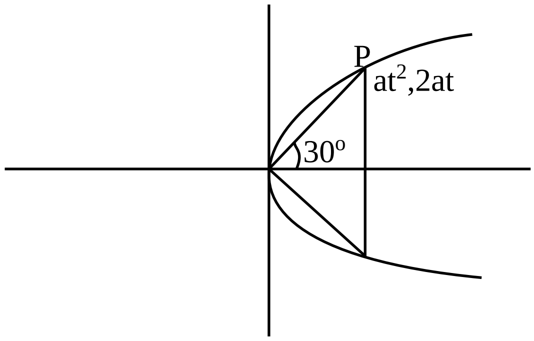
$$=\frac{\frac{\lambda}{\lambda+4}}{\frac{\lambda}{\lambda+4}+\frac{9}{10}}$$

$$\Rightarrow \frac{10\lambda}{10\lambda+9(\lambda+4)}=\frac{4}{10}$$

$$\Rightarrow 100\lambda=40\lambda+36\lambda+144$$

$24\lambda = 144$

$\lambda = 6$



$m = \frac{2}{t}=\frac{1}{\sqrt{3}}$

$t = 2\sqrt{3}$

$P\left(12a, 4\sqrt{3}a\right)$

$(\text{Side})^2 = 144a^2 + 48a^2$

$= 192\times\frac{9}{4} = 432$

83. Let $S = \{\theta \in [0,2\pi): \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta\in s}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to

Sol. **2**

$\tan(\pi\cos\theta)=\tan[-\pi\sin\theta]$

$\pi\cos\theta = n\pi - \pi\sin\theta \qquad (n\in I)$

$\cos\theta + \sin\theta = n$

$n\in\left[-\sqrt{2},\sqrt{2}\right] \qquad n\in\{-1,0,1\}$

$\cos\left(\theta-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}, \qquad \cos\left(\theta-\frac{\pi}{4}\right)=0, \qquad \cos\left(\theta-\frac{\pi}{4}\right)=\frac{-1}{\sqrt{2}}$

$\theta-\frac{\pi}{4}=2m\pi\pm\frac{\pi}{4}, \theta-\frac{\pi}{4}=2m\pi+\frac{\pi}{2}, \qquad \theta-\frac{\pi}{4}=2m\pi\pm\frac{3\pi}{4}$

$\theta=2m\pi+\frac{\pi}{2}, \qquad \theta=2m\pi+\frac{3\pi}{4}, \qquad \theta=2m\pi+\pi$

$\theta=2m\pi, \qquad \theta=2m\pi-\frac{\pi}{2}$

$\theta=\left\{\frac{\pi}{2},0,\cancel{\frac{\pi}{4}},\pi,\frac{3\pi}{2},\frac{3\pi}{4},\frac{7\pi}{4}\right\}$

$$\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}$$

$$\therefore \sum\sin^2\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2}+\frac{1}{2}+\cancel{1}+\frac{1}{2}+\frac{1}{2}=2 \text{ Ans.}$$

84. If $\frac{1^3+2^3+3^3+\cdots \text{ up to n terms}}{1\cdot3+2\cdot5+3\cdot7+\cdots \text{ up to n terms}}=\frac{9}{5}$, then the value of n is

Sol. **5**

$$\frac{\left(\frac{n(n+1)}{2}\right)^2}{\sum r(2r+1)}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3}+\frac{1}{2}} \Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+5)}{6}}=\frac{9}{5}$$

$$\Rightarrow \frac{3(n+1)n}{2(4n+5)}=\frac{9}{5}$$

$\Rightarrow 5n^2 + 5n = 24n + 30$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$5n^2 - 25n + 6n - 30 = 0$

$(5n + 6)(n - 5) = 0$

$n = 5$

85. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n, x \neq 0. n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is

Sol. **405**

$^nC_0 - {}^nC_1\cdot 3 + {}^nC_2\cdot 3^2$

$1 - 3n + \frac{9n(n-1)}{2}=376$

$2 - 6n + 9n^2 - 9n = 752$

$9n^2 - 15n - 750 = 0$

$3n^2 - 5n - 250 = 0$

$3n^2 - 30n + 25n - 250 = 0$

$(3n + 25)(n - 10) = 0$

$n = 10$

$\therefore T_{r+1} = {}^{10}C_r (x)^{10-r}\left(\frac{-3}{x^2}\right)^r$

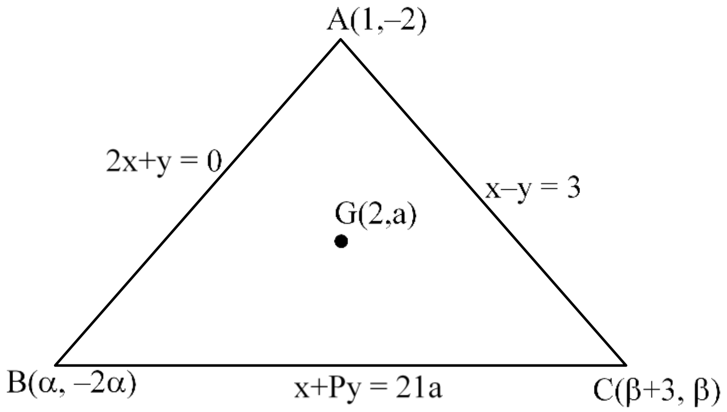
$x^{10-3r} = x^4 \Rightarrow 3r = 6$

$r = 2$

$\therefore T_3 = {}^{10}C_2 \times 3^2 \Rightarrow \frac{10\times 9}{2}\times 9 = 405$

86. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0, x + py = 21a, (a \neq 0)$ and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of $\triangle ABC$. Then $(BC)^2$ is equal to

Sol. **122**



$\frac{\alpha+\beta+4}{3} = 2 \qquad \frac{-2\alpha-2+\beta}{3} = a$

$\alpha + \beta = 2 \qquad -2\alpha + \beta = 3a + 2$

$-2\alpha + 2 - \alpha = 3a + 2$

$\alpha = -a$

put 'B' in BC

$\alpha - 2p\alpha = 21a$

$\alpha.(1 - 2p) = 21a \qquad 2p - 1 = 21$

$p = 11$

put 'C' in BC

$\beta + 3 + 11\beta = 21a$

$21\alpha + 12\beta + 3 = 0$

also $\beta = 2 - \alpha \qquad$ Solving $\alpha = -3, \beta = 5$

$\therefore BC = \sqrt{122}$

$BC^2 = 122$

87. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0, \vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

Sol. **8**

$\bar{a} = (1, 2, \lambda) \qquad \bar{b} = (3, -5, -\lambda)$

$\bar{a}\cdot\bar{c}=7$, $\qquad \bar{b}\cdot\bar{c}=\frac{-43}{2}$,

$(\bar{a}-\bar{b})\times\bar{c}=0$

$c=x[-2,7,2\lambda]=(-2x,7x,2\lambda x)$

now, $\quad \bar{a}.\bar{c}=-2x+14x+2\lambda^2x=7$

$$2\lambda^2x+12x=7 \qquad ...(1)$$

$\bar{b}\cdot\bar{c}=-6x-35x-2\lambda^2x=\frac{-43}{2}$

$$-41x-2\lambda^2x=\frac{-43}{2} \qquad ...(2)$$

by adding (1) + (2)

$$-29x=\frac{-29}{2}\Rightarrow x=\frac{1}{2}$$

$\therefore \lambda^2+6=7 \quad \Rightarrow \lambda^2=1$

$|\bar{a}\cdot\bar{b}|=|3-10-\lambda^2|=|-8|$

88. The minimum number of elements that must be added to the relation $R=\{(a,b),(b,c),(b,d)\}$ on the set $\{a,b,c,d\}$ so that it is an equivalence relation, is

Sol. **13**

	a	b	c	d
a	1	✓	8	9
b	5	2	✓	✓
c	10	6	3	11
d	12	7	13	4

1, 2, 3, 4 $\rightarrow$ for reflexive

5, 6, 7 $\rightarrow$ for symmetric

8, 9, 10, 11, 12, 13 $\rightarrow$ for transitive

89. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8 A is equal to

Sol. **36**

$y^2-2y+1=-x+1 \qquad x+y=0$

$(y-1)^2=-(x-1) \qquad x+1+y+1=0$

$y^2=-4Ax \qquad x+y+2=0$

$y=y-1$

$x=x-1$

$y^2=-x$

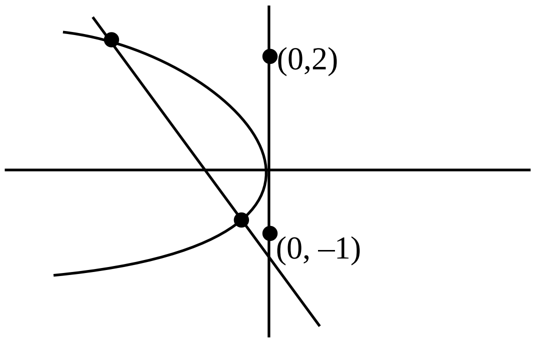
$x+y=-2$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, y = -1$



$$\text{Area} = \int_{-1}^{2} \left(-y^2\right) - (-2 - y)\,dy$$

$$= \left(\frac{-y^3}{3} + 2y + \frac{y^2}{2}\right)_{-1}^{2}$$

$$= \left(\frac{-8}{3} + 4 + 2\right) - \left(\frac{1}{3} - 2 + \frac{1}{2}\right)$$

$$= \frac{-8}{3} + 6 - \frac{1}{3} + 2 - \frac{1}{2}$$

$$A = -3 + 8 - \frac{1}{2} \Rightarrow \frac{9}{2}$$

$\therefore\ 8A = 36$

90. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

Sol. $f'(x) + f(x)\cdot\sqrt{1 - \log^2 \cdot f(x)} = 0$

$$\frac{dy}{dx} = -y\sqrt{1 - \log^2 y}$$

$f(0) = e$

$$\frac{dy}{y\sqrt{1 - \log^2 y}} = -dx$$

$\log y = t$

$$\frac{1}{y}dy = dt$$

$$\frac{dt}{\sqrt{1 - t^2}} = -dx$$

$\operatorname{Sin}^{-1}(t) = -x + C$

$\operatorname{Sin}^{-1}[\log y] = -x + C$

$x = 0 \quad \text{Sin}^{-1}(1) = C \Rightarrow \frac{\pi}{2}$

$\log(y) = \text{Sin}\left(\frac{\pi}{2} - x\right)$

$x = \frac{\pi}{6} \quad \log_e\left[f\left(\frac{\pi}{6}\right)\right] = \text{Sin}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\therefore \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27$